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**G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.**



**UG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.**

(For those admitted in June 2021 and later)

**PROGRAMME AND BRANCH: B.Sc., MATHEMATICS**

| SEM | CATEGORY | COMPONENT | COURSE CODE | COURSE TITLE     |
|-----|----------|-----------|-------------|------------------|
| VI  | PART-III | CORE      | U21MA613    | COMPLEX ANALYSIS |

**Date & Session: 24.04.2025/FN**

**Time : 3 hours**

**Maximum: 75 Marks**

| Course Outcome | Bloom's K-level | Q. No. | SECTION – A (10 X 1 = 10 Marks)<br>Answer <u>ALL</u> Questions.  |
|----------------|-----------------|--------|--|
| CO1            | K1              | 1.     | If $f(z)=z^3$ , $z=x+iy$ then the value of $v(x,y)$ is.<br>a) $x^3-3xy^2$ b) $3x^2y-y^3$ c) $3xy^2-y^3$ d) $3x^2y-x^3$   |
| CO1            | K2              | 2.     | The Complex form of CR equation is.<br>a) $f_x=if_y$ b) $f_y=-if_x$ c) $f_x=-if_y$ d) $f_x=f_y$  |
| CO2            | K1              | 3.     | Any bilinear transformation with only one finite fixed point is called.<br>a) Parabolic                      b) Hyprbolic                      c) Elliptic                      d) None of these                           |
| CO2            | K2              | 4.     | The transformation $w= \bar{z}$ is.<br>a) bilinear transformation                      b) not a bilinear transformation<br>c) inverse transformation                      d) None of these                                 |
| CO3            | K1              | 5.     | The value of $\int_C \frac{dz}{z-3}$ where C is the circle $ z-2 =5$ .<br>a) 2                      b) -2                      c) 0                      d) 1  |
| CO3            | K2              | 6.     | The value of $\frac{1}{2\pi i} \int_C \frac{z^2+5}{z-3} dz$ where C is $ z =4$ is.<br>a) 15                      b) 14                      c) 10                      d) 0  |
| CO4            | K1              | 7.     | The Taylors series expansion of $f(z)$ about the point zero is called the.<br>a) Laurents series                      b) Maclaurins series<br>c) Power series                      d) None of these                        |
| CO4            | K2              | 8.     | The zero of $f(z)=\sin z$ is.<br>a) 1                      b) 2                      c) 0                      d) 3  |
| CO5            | K1              | 9.     | The poles of $f(z) = \frac{z^2}{(z-2)(z-3)}$ .<br>a) 2,3                      b) -2,3                      c) 2,-3                      d) -2,-3   |
| CO5            | K2              | 10.    | The residue of $f(z)=\frac{e^z}{z^2}$ is.<br>a) 0                      b) 1                      c) 2                      d) 3  |
| Course Outcome | Bloom's K-level | Q. No. | SECTION – B (5 X 5 = 25 Marks)<br>Answer <u>ALL</u> Questions choosing either (a) or (b)   |
| CO1            | K3              | 11a.   | State and Prove complex form of CR equations.<br>(OR)  |
| CO1            | K3              | 11b.   | If $\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$ prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$ |

|     |    |      |   |
|-----|----|------|---|
| CO2 | K3 | 12a. | Prove that any bilinear transformation preserves cross ratio.<br><b>(OR)</b>  |
| CO2 | K3 | 12b. | Find the invariant points of the transformation.<br>i. $w = \frac{1+z}{1-z}$ ii. $W = \frac{1}{z-2i}$   |
| CO3 | K4 | 13a. | Evaluate $\int_C \bar{z} dz$ from $z=0$ to $z=4+2i$ along the curve C consisting of the line segment from $z=0$ to $z=2i$ followed by the line segment from $z=2i$ to $z=4+2i$ .<br><b>(OR)</b> |
| CO3 | K4 | 13b. | Evaluate $\int_C \frac{zdz}{z^2-1}$ where C is the positively oriented circle $ z =2$ .   |
| CO4 | K4 | 14a. | Expand $\cos z$ into a Taylor's series about the point $z = \frac{\pi}{2}$ and determine the region of convergence.<br><b>(OR)</b>  |
| CO4 | K4 | 14b. | Find the Laurent's series for $\frac{z}{(z+1)(z+2)}$ about $z=-2$ .   |
| CO5 | K5 | 15a. | Find the poles of $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$ and determine the residues at the poles.<br><b>(OR)</b>  |
| CO5 | K5 | 15b. | Prove that $\int_C \frac{e^{2z}}{(z+1)^3} dz = \frac{4\pi i}{e^2}$ where C is $ z  = \frac{3}{2}$ .   |

| Course Outcome | Bloom's K-level | Q. No. | <p align="center"><b>SECTION – C (5 X 8 = 40 Marks)</b><br/> <b>Answer ALL Questions choosing either (a) or (b)</b></p>  |
|----------------|-----------------|--------|--|
| CO1            | K3              | 16a.   | Show that $u(x,y) = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic. Find an analytic function $f(z)$ in terms of $z$ with the given $u$ for its real part.<br><b>(OR)</b>  |
| CO1            | K3              | 16b.   | Given the function $w = z^3$ where $w = u + iv$ . Show that $u$ and $v$ satisfy the CR equations. Prove that the families of curves $u = c_1$ and $v = c_2$ ( $c_1$ and $c_2$ are constants) are orthogonal to each other.   |
| CO2            | K4              | 17a.   | Determine the bilinear transformation which maps $0, 1, \infty$ into $i, -1, -i$ respectively. Under this transformation show that the interior of the unit circle of the $z$ plane maps onto the half plane left to the $v$ axis (left half of the $w$ plane).<br><b>(OR)</b> |
| CO2            | K4              | 17b.   | Prove that a bilinear transformation $w = \frac{az+b}{cz+d}$ where $ad-bc \neq 0$ maps the real axis into itself iff $a, b, c, d$ are real.  |
| CO3            | K4              | 18a.   | Show that $\int_C  z ^2 dz = -1+i$ where C is the square with vertices $O(0,0), A(1,0), B(1,1)$ and $C(0,1)$ .<br><b>(OR)</b>  |
| CO3            | K4              | 18b.   | State and Prove Cauchy's theorem.  |
| CO4            | K5              | 19a.   | State and Prove Taylor's series.<br><b>(OR)</b>  |
| CO4            | K5              | 19b.   | Find the Laurent's series expansion of the function $\frac{z^2-1}{(z+2)(z+3)}$ valid in the annular region $2 <  z  < 3$ .   |
| CO5            | K5              | 20a.   | State and Prove Cauchy's Residue theorem.<br><b>(OR)</b>   |
| CO5            | K5              | 20b.   | Evaluate, using i. Cauchy's integral formula ii. Residue theorem $\int_C \frac{z+1}{z^2+2z+4} dz$ where C is the Circle $ z+1+i  = 2$ .  |